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NDC Dynamic Equilibrium model with financial and demographic risks

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ABSTRACT

Classical social security pension schemes, combining a defined benefit philosophy and a pay as you go system, are clearly under threat taking into account the general demographic evolution of many countries for the next decades. An interesting attempt to solve this problem is to maintain the pay as you go mechanism but moving to a defined contribution system (notional accounts or NDC schemes). In order to implement such schemes it is necessary to define various parameters such as the notional rate, the annuity conversion price or the indexation procedure. All these choices are not neutral in term of stability of the system. The purpose of this paper is to present a 3 generations-model permitting to model the influence of the dynamic evolution of the financial and the demographic parameters on the equilibrium of a NDC system.

KEYWORDS

Social Security, Notional account, Demographic Risk, Longevity, Actuarial Fairness

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1. INTRODUCTION

Population ageing and decreasing of fertility have generated need to reform social security systems of pension based on a "PAYG" technique. In deed the combination of a defined benefit philosophy with a PAYG technique leads clearly to a major financial disaster in case of demographic crisis.

A first possible solution to this future challenge is to switch from PAYG to a funded regime where actuarial equilibrium is a natural consequence of the mechanism. But shifting for a social security system from PAYG to full funding is not so simple and can also lead to dangerous accumulation of financial risk.

Another promising reform seems to be the NDC scheme, which has recently been adopted in various countries as Italy, Latvia, Poland and Sweden.

Several researches [Valdes Prieto (1999), Scherman (1999) and Auerbach and Lee (2007)] have shown however that NDC mechanism, as applied in practice, does not imply directly equilibrium in a context of financial and demographic uncertainty. More precisely, present conditions of applications of NDC cannot guarantee automatic financial stability with demographic and longevity risk.

If the automatic financial stability is the ability of a pension plan to adjust to financial shocks without legislative intervention (Valdés-Prieto, 1999), we will show in this work how the automatic financial stability is possible in the NDC in a model with different generations and taking account financial, demographic and longevity risks in a deterministic framework.

The paper is organized as follows. In section 2, we formalize the demographic model with 3 generations and compute in a static environment the NDC equilibrium. Then section 3 moves to a dynamic model where the 3 main demographic and financial processes can change on time. In section 3 we compute an equilibrated NDC scheme assuming a same amount of pension for all generations of retirees. Section 4 generalizes this approach by considering generation pension. In particular, we present various equilibrated schemes based on different choices of the 3 main parameters of a NDC system. Non equilibrated choices are also presented. Section 5 concludes the paper.

2. STATIC MODEL WITH 3 GENERATIONS

2.1. INTRODUCTION

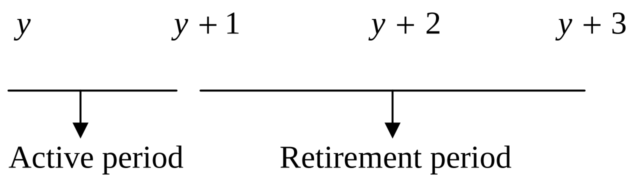
We will consider here a dynamic model of population based on 3 generations: one generation of active people and two generations of retired people. Traditionally, it is more usual to work only with 2 generations in an overlapping model of pension [see for instance Vermeylen (2007)]. Nevertheless, in the context of NDC and in order to capture the effect of indexation of pension after retirement (this effect being one of the key parameters in a NDC scheme), it appears essential to introduce at least two generations of retirees.

So we suppose that at age y people are entering the population as active member and receive a salary, on which a pension contribution is computed.

At age $y + 1$, people are retired and receive their first pension amount.

At age $y + 2$, the survivors receive a second pension amount, based on an indexation rule after retirement.

At age $y + 3$, there is no survivor in this generation and no other payment occurs.



In this population, we introduce a NDC pension plan. The purpose of this section is to look at equilibrium conditions based on the parameters of the environment.

2.2. NOTATIONS AND ASSUMPTIONS

We introduce the following demographic elements:

$L(x, t)$: number of people present at age x at time t $(x = y, y+1, y+2)$

$E(t) = L(y, t)$: number of people entering the population at time t

We ignore mortality between age y and age $y + 1$ like in the classical NDC paradigm where mortality is not taken into account in the active part of the career.

Evolution of the active population: if δ is the constant annual rate of increase of the entrance function of the population, the people entering the population at time t are given by:

$$E(t) = E(0).e^{\delta t}$$

Evolution of the individual wages: Let us define γ the annual rate of increase of the salary, the individual wages at time t

$$s(t) = s(0).e^{\gamma t}$$

Evolution of the global wage mass:

$$S(t) = S(0).e^{(\delta+\gamma)t}$$

Survival probability between age y and age $y+1$: p

Individual contribution: the contribution rate to the NDC plan is constant, so we have:

$$c(t) = \pi.s(t)$$

We are working with 3 important parameters: increase of individual salary, growth of the population and longevity effect through the survival probability. Taking into account the stationary assumptions of this model, we assume that the 2 generations of retirees will receive at time t the same amount pension $P(t)$. This assumption will be relaxed later when we will consider dynamic financial and demographic models.

2.3. EQUILIBRIUM CONDITION

We will apply on this model the classical rule of pay as you go (PAYG):

Income of the year = Outcome of the year

starting from the contributions and computing the level of pension generated by this PAYG arrangement. Then we will interpret the amount of pension in terms of a notional account formula.

1°) Income for the year t: $C(t) = \pi S(0).e^{\gamma t}.e^{\delta t}$

2°) Outcome for the year t:

Total number of pensioners: $L(t) = E_0.(e^{\delta(t-1)} + p.e^{\delta(t-2)})$

So the equilibrium relation of PAYG becomes now: $P(t).L(t) = C(t)$

We get for the pension amount to pay:

$$
\begin{aligned}
\mathrm{P(t)} &= \frac{\pi.\mathrm{S(0)}.e^{\gamma t}.e^{\delta t}}{\mathrm{E(0)}.(e^{\delta(t-1)} + p e^{\delta(t-2)})} = \frac{\pi.s(0).e^{\gamma t}.e^{\delta}}{1 + p\,e^{-\delta}} \\
&= \frac{\pi.s(0).e^{\gamma(t-1)}.e^{\gamma+\delta}}{1 + p\,e^{-\delta}}
\end{aligned}
\tag{2.1}
$$

The replacement rate $RR(t)$ is given by:

$$RR(t) = \frac{P(t)}{s(t)} = \pi \frac{e^{\delta}}{(1 + p\,e^{-\delta})}$$

which is an increasing function of the demographic rate δ, of the contribution rate π and a decreasing function of the survival probability p

This formula of pension can be seen as a notional account formula. The pension $P(t)$ is paid at people just retired who were active people at time $t-1$. If we apply the traditional notional account formula, it comes:

$$Pension = \frac{contribution \quad x \quad revalorization}{annuity} = \frac{c(t-1).R(t)}{a(t)} \tag{2.2}$$

The contribution is equal to: $c(t-1) = \pi.s(0).e^{\gamma(t-1)}$

So a natural identification to a NDC design can be described as follows:

1°) the revalorization is made using the total growth of salaries: $R(t) = e^{\gamma+\delta}$

2°) the annuity is also constant and given by: $a(t) = 1 + p\ e^{-\delta}$ (2.3)

This particular choice induces a special form for the indexation process. For this we have to compare $P(t-1)$ with $P(t)$ knowing that $P(t-1)$ is given by:

$$P(t-1) = \frac{\pi . s(0) . e^{\gamma(t-2)} . e^{\gamma+\delta}}{(1+ p e^{-\delta})}$$

So if we compute the ratio $P(t)/\ P(t-1)$ we obtain the rule of indexation in equilibrium:

$$RP(t) = \frac{P(t)}{P(t-1)} = e^{\gamma} \qquad (2.4)$$

2.4. OPTIMAL DESIGN OF A NDC SCHEME

At this stage, in static conditions, the conclusion in a deterministic environment can be summarized as follows: if we want an exact equilibrium in NDC, we can choose the following parameters:

- **Revalorization of the salaries before retirement to compute the first pension:** total growth of salaries (individual growth + growth of population)
- **Pension indexation after retirement**: individual growth of salaries
- **Computation of the annuities**: with a discount rate equal to the population growth and using the survival probability.

This particular design is generally called "*the canonical choice*". But other combinations of the 3 processes are possible. For instance we could adopt a same value for the coefficient of revalorization of salaries and the indexation of pensions, corresponding to the increase of salaries:

$$RP(t) = R(t) = e^{\gamma} \qquad (2.5)$$

Then relation (2.2) gives the compatible value of the annuity:

$$a(t) = e^{-\delta} + p e^{-2\delta} \qquad (2.6)$$

In general, in this model the value of the pension indexation process is fixed and given by formula (2.4). The other processes $R(t)$ and $a(t)$ can be chosen more arbitrarily but consistently with constraint (2.4) and will generate finally the same pension.

If we choose a general revalorization process given by:

$$R(t) = e^{\alpha}$$

then the annuity conversion must be equal to:

$$a(t) = e^{\alpha-\gamma-\delta}(1+ p e^{-\delta})$$

3. DYNAMIC MODEL WITH UNIFORM PENSION

3.1. ASSUMPTIONS

The basic model proposed in section 2 was based on constant and steady conditions for the financial and demographic parameters. Here we consider that these parameters can fluctuate from one year to another. More precisely the growth of the active population denoted by δ and the growth of individual wages denoted by γ are now functions of time:

δ_j : growth rate of the active population for year j (j=1,2,…)

γ_j : growth rate of individual wages for year j (j=1,2,…)

The survival probability p can also change from one generation to another: p_j

In this section we will still assume that the 2 generations of retires will receive at time t a same amount of pension $P(t)$ (solidarity between the retirees). This assumption, natural in section 2 taking into account the stability of the parameters, can now be discussed and will be relaxed in section 4.

The **INCOME** for the year t can now be computed as follows:

Evolution of the active population $\quad E(t) = E(0).\exp\left(\sum_{i=1}^{t}\delta_i\right)$

Evolution of the individual wages $\quad s(t) = s(0).\exp\left(\sum_{i=1}^{t}\gamma_i\right)$

Evolution of the global wages $\quad S(t) = E(t).s(t) = S(0).\exp\left(\sum_{i=1}^{t}\delta_i + \gamma_i\right)$

Survival probability $\quad p_t$

Individual contribution for the year t $\quad c(t) = \pi.s(0).\exp\left(\sum_{i=1}^{t}\gamma_i\right)$

Total contribution for the year t $$C(t) = \pi.E(0).s(0).\exp\left(\sum_{i=1}^{t}\delta_i + \gamma_i\right)$$

The **OUTCOME** for the year t is given by:

Total number of pensioners $$L(t) = L_1(t) + L_2(t)$$

New generation of retired people $$L_1(t) = E(0).\exp\left(\sum_{i=1}^{t-1}\delta_i\right)$$

Generation retired the year before $$L_2(t) = E(0).\exp\left(\sum_{i=1}^{t-2}\delta_i\right).p_t$$

Pension for the new and old retired generations $P(t)$

3.2. EQUILIBRIUM CONDITION

Using the same equilibrium mechanism as in section 2.3, we get in this dynamic model the following expression for the amount of pension $P(t)$ generalizing formula (2. 1):

$$P(t) = \pi.s(0).\exp(\sum_{i=1}^{t-1}\gamma_i).\frac{\exp(\gamma_t + \delta_t)}{1 + p_t.\exp(-\delta_{t-1})} \qquad (3.1)$$

The replacement rate becomes:

$$RR(t) = \frac{P(t)}{s(t)} = \pi\frac{e^{\delta_t}}{(1 + p_t\, e^{-\delta_{t-1}})}$$

Once again this value can be interpreted as a notional account formula. In order to define a NDC pension schemes we must choose 3 processes:

- the coefficient of revalorization of the notional account $R(t)$
- the annuity $a(t)$
- the coefficient of indexation of the pensions $RP(t)$

Then the generated pension is given in our 3 period model with uniform pension by the 2 following expressions, the first one based on the notional conversion at retirement and the second one linking 2 successive pension amounts with the indexation rule:

$$P(t) = \frac{c(t-1).R(t)}{a(t)} \tag{3.2}$$

$$P(t) = P(t-1)RP(t) \tag{3.3}$$

Clearly we have an infinite number of possibilities of choice for the processes $R(t)$, $a(t)$ and $RP(t)$ in order to replicate the value given by (3.1) remaining consistent with the NDC mechanism (3.2) and (3.3).

A first natural choice coherent with section 2 ("*canonical choice*") and directly inspired by (3.1) leads to the following identification:

3.2.1. First design (*canonical design*)

We take as coefficient of revalorization the numerator of (3.1) and as annuity the denominator of (3.1):

$$\begin{aligned} R(t) &= \exp(\gamma_t + \delta_t) \\ a(t) &= 1 + p_t \exp(-\delta_{t-1}) \end{aligned} \tag{3.4}$$

The revalorization of the salaries is made using the total growth of salaries. The annuity is computed using as discount rate, the population growth of the past (and not of the current year!) and as survival probability, the probability as observed the year of computation (and not a projected value for the considered retiree generation!). Using formula (3.3) we obtained the consistent coefficient of indexation of pension

$$RP(t) = \exp(\gamma_t).\frac{\exp(\delta_t)}{\exp(\delta_{t-1})}.\frac{1 + p_{t-1}e^{-\delta_{t-2}}}{1 + p_t e^{\delta_{t-1}}} = \exp(\gamma_t).\frac{\exp(\delta_t)}{\exp(\delta_{t-1})}.\frac{a(t-1)}{a(t)} \tag{3.5}$$

If the demographic conditions are static (growth of the population and longevity), then this system permits an indexation of the pension in line with the salary increase:

$$RP(t) = \exp(\gamma_t)$$

This equivalence disappears in a dynamic demographic model.

For instance, if the demographic growth δ stays constant but if the longevity is increasing, then:

$$RP(t) = \exp(\gamma_t).\frac{1 + p_{t-1}e^{-\delta}}{1 + p_t e^{\delta}} < \exp(\gamma_t)$$

3.2.2. Second design (*uniform indexation before and after retirement*)

Once again, exactly as in section 2, the "*canonical choice*" generates two different processes of indexation (respectively $R(t)$ before retirement given by (3.4) and $RP(t)$ after retirement given by (3.5)).

We could alternatively ask a same value for these two indexation processes (same philosophy as formula (2.5)). Because the value of $RP(t)$ is fixed by the relations (3.3) and (3.1), we are only free to change the value of the process $R(t)$.

The choice leads so to:

$$R(t) = RP(t) = \exp(\gamma_t).\frac{\exp(\delta_t)}{\exp(\delta_{t-1})}.\frac{1 + p_{t-1}e^{-\delta_{t-2}}}{1 + p_t e^{\delta_{t-1}}}$$

Then the equilibrium formula (3.2) gives the value of the annuity:

$$a(t) = e^{-\delta_{t-1}} + p_{t-1}\, e^{-(\delta_{t-1}+\delta_{t-2})}$$

3.2.3. Third design (*revalorization in line with salaries*)

We could also choose to use a revalorization process in line with the growth of individual salaries (cf. (2.5) in the static case):

$$R(t) = \exp(\gamma_t)$$

The pension indexation is still given then by (3.5) and the annuity becomes now:

$$a(t) = e^{-\delta_t} + p_t\, e^{-(\delta_t+\delta_{t-1})}$$

Let us remark that in this model (… exactly as in section 2) these different designs are just for identification and interpretation purposes but generate finally the same pension liabilities given at time t by $P(t)$ for all generations [pension amount given in the 3 designs by formula (3.1)]. The situation will be quite different in the next section.

4. DYNAMIC MODEL WITH GENERATION PENSION

4.1. ASSUMPTION

Instead of assuming as in section 3 a uniform amount of pension for all the retirees at a same moment, we will now move to a generation pension model. In a dynamic model, the 2 generations have a different history (and also a different future through changing survival probabilities); so it is now surprising to give them different amounts of pension.

We will use the following notations:

Pension for the new retired generations at time t : $P_1(t)$

Pension for the old retired generations at time t : $P_2(t)$

4.2. EQUILIBRIUM CONDITION

Having two different levels of pension but only one equilibrium relation (the classical PAYG budget constraint), we must impose an additional condition in order to define the scheme. For this, we will start from a natural NDC formulation for the new pensions.

4.2.1. Calculation of the pension for the new retirees

The pension of the new retirees is computed applying the pure notional account calculus:

$$P_1(t) = \frac{c(t-1).R(t)}{a(t)}$$

or:

$$P_1(t) = \pi.s(0).\exp\left(\sum_{i=1}^{t-1}\gamma_i\right).\frac{R(t)}{a(t)} \tag{4.1}$$

where the revalorization process R(t) and the annuity a(t) must be defined by the system.

4.2.2. Calculation of the pension for the old generation, the year before

Similarly this pension as been computed the year before following the same logic:

$$P_1(t-1) = \pi.s(0).\exp\left(\sum_{i=1}^{t-2}\gamma_i\right).\frac{R(t-1)}{a(t-1)} \tag{4.2}$$

4.2.3. Calculation of the pension for the old generation in t

In order to compute the pension for the old generation in t, we use the PAYG equilibrium condition:

$$L_1(t).P_1(t)+L_2(t).P_2(t)=C(t).$$

So the amount $P_2(t)$ is given explicitly by:

$$P_2(t)=\frac{C(t)-L_1(t).P_1(t)}{L_2(t)} \qquad \text{and} \qquad \frac{P_2(t)}{P_1(t)}=\frac{C(t)}{L_2(t)}.\frac{1}{P_1(t)}-\frac{L_1(t)}{L_2(t)}.$$

$$\frac{P_2(t)}{P_1(t)}=\frac{\pi.E(0).s(0).\exp\left(\sum_{i=1}^{t}\delta_i+\gamma_i\right)}{E_0.\exp\left(\sum_{i=1}^{t-2}\delta_i\right).p_t}.\frac{a(t)}{\pi.s(0).\exp\left(\sum_{i=1}^{t-1}\gamma_i\right).R(t)}-\frac{E_0.\exp\left(\sum_{i=1}^{t-1}\delta_i\right)}{E_0.\exp\left(\sum_{i=1}^{t-2}\delta_i\right).p_t}$$

$$=\frac{\exp(\delta_{t-1}+\delta_t+\gamma_t)}{p_t}.\frac{a(t)}{R(t)}-\frac{\exp(\delta_{t-1})}{p_t}$$

Finally we obtain

$$\frac{P_2(t)}{P_1(t)}=\frac{\exp(\delta_{t-1})}{p_t}.\left[\exp(\delta_t+\gamma_t).\frac{a(t)}{R(t)}-1\right] \tag{4.3}$$

This ratio compares the level of pension for the 2 generations of retirees at a same moment. In sections 2 and 3 this ratio was by definition equal to 1.

4.2.4. Calculation of the indexation process

We can also compute the indexation process for the old generation; after computation, it comes:

$$RP(t)=\frac{P_2(t)}{P_1(t-1)}=\frac{\exp(\delta_{t-1}+\gamma_{t-1})}{p_t}.\frac{R(t)}{R(t-1)}.\frac{a(t-1)}{a(t)}.\left[\exp(\delta_t+\gamma_t)\frac{a(t)}{R(t)}-1\right] \tag{4.4}$$

4.3. PARTICULAR CHOICES OF EQUILIBRATED NDC PARAMETERS

In order to define one particular equilibrated scheme we must choose values for the 3 basic processes $R(t)$, $RP(t)$ and $a(t)$ such as to fulfill relation (4.4). There are an infinite number of such possible equilibrated NDC systems. Here are some possible coherent schemes. Other possibilities can be off course considered! Let us remark that the different choices of parameters are here more than just an identification exercise as in section 3; they generate now different values of pension and especially a different ratio of pension between the two generations of retirees.

We will present six models which can be summarized in the following table:

	Prospective probability	Present probability
Basic case	Model 1	Model 4
Life expectancy	Model 2	Model 5
Salary Indexation	Model 3	Model 6

4.3.1. Models with prospective probabilities

We first consider 3 models where the annuity is computed using prospective probabilities.

4.3.1.1. Model 1 (*canonical choice*)

1°) the revalorization at the total growth of salaries: $R(t) = \exp(\gamma_t + \delta_t)$

2°) the annuity is given by: $a(t) = 1 + p_{t+1}.\exp(-\delta_t)$

3°) the indexation of pension is solution of the equilibrium relation (4.4).

Then the replacement rate for the new generation of retirees is given by:

$$RR_1(t) = \frac{P_1(t)}{s(t)} = \pi \frac{e^{\delta_t}}{(1 + p_{t+1}\, e^{-\delta_t})}$$

The pension for the old retirees becomes:

$$\frac{P_2(t)}{P_1(t)} = \frac{\exp(\delta_{t-1})}{p_t} . \left[\exp(\delta_t + \gamma_t) . \frac{1 + p_{t+1}.\exp(-\delta_t)}{\exp(\gamma_t + \delta_t)} - 1 \right] = \frac{p_{t+1}}{p_t} \frac{e^{\delta_{t-1}}}{e^{\delta_t}} \tag{4.5}$$

For the old generation, the indexation process $RP(t)$ is given by :

$$RP(t) = \frac{P_2(t)}{P_1(t-1)} = \exp(\gamma_t) . \frac{p_{t+1}}{p_t} \frac{1 + p_{t-1}.\exp(-\delta_{t-1})}{1 + p_t.\exp(-\delta_t)} = \exp(\gamma_t) . \frac{p_{t+1}}{p_t} \frac{a(t-1)}{a(t)} \tag{4.6}$$

The natural indexation by the growth of salary is corrected by the ratio of two successive annuities and by the ratio of two successive survival probabilities.

4.3.1.2. Model 2 (*life expectancy*)

1°) the revalorization at the total growth of salaries $R(t) = \exp(\gamma_t + \delta_t)$

2°) the annuity is given by the life expectancy: $a(t) = 1 + p_{t+1}$

The pension for the old generation is now:

$$\frac{P_2(t)}{P_1(t)} = \frac{\exp(\delta_{t-1})}{p_t} \cdot \left[\exp(\delta_t + \gamma_t) \cdot \frac{1 + p_{t+1}}{\exp(\gamma_t + \delta_t)} - 1 \right] = \frac{p_{t+1}}{p_t} e^{\delta_{t-1}} \tag{4.7}$$

The indexation process becomes:

$$RP(t) = \frac{P_2(t)}{P_1(t-1)} = \exp(\delta_t + \gamma_t) \cdot \frac{p_{t+1}}{p_t} \frac{a(t-1)}{a(t)} \tag{4.8}$$

4.3.1.3. Model 3 (*revalorization with salary growth*)

1°) the revalorization at the individual growth of salaries: $R(t) = \exp(\gamma_t)$

2°) the annuity is given by: $a(t) = 1 + p_{t+1} e^{-\delta_t}$

The pension for the old generation is now:

$$\frac{P_2(t)}{P_1(t)} = \frac{\exp(\delta_{t-1})}{p_t} (\exp(\delta_t) + p_{t+1} - 1) \tag{4.9}$$

The indexation process has the following more complicated form:

$$RP(t) = \frac{P_2(t)}{P_1(t-1)} = \exp(\delta_{t-1} + \gamma_t) \frac{a(t-1)}{a(t)} \frac{(e^{\delta_t} + p_{t+1} - 1)}{p_t} \tag{4.10}$$

4.3.2. Models with present probabilities

The 3 models presented just before can be developed with annuities computed with present survival probabilities instead of prospective probabilities; the 3 cases considered just before becomes then:

4.3.2.1. Model 4 (*canonical choice*)

1°) the revalorization at the total growth of salaries: $R(t) = \exp(\gamma_t + \delta_t)$

2°) the annuity is given by: $a(t) = 1 + p_t . \exp(-\delta_t)$

3°) the indexation of pension is solution of the equilibrium relation (4.4).

The pension for the old retirees becomes:

$$\frac{P_2(t)}{P_1(t)} = \frac{\exp(\delta_{t-1})}{p_t} \cdot \left[\exp(\delta_t + \gamma_t) \cdot \frac{1 + p_t . \exp(-\delta_t)}{\exp(\gamma_t + \delta_t)} - 1 \right] = \frac{e^{\delta_{t-1}}}{e^{\delta_t}} \tag{4.11}$$

The indexation process $RP(t)$ is given by :

$$RP(t)=\frac{P_2(t)}{P_1(t-1)}=\exp(\gamma_t).\frac{1+p_{t-1}.\exp(-\delta_{t-1})}{1+p_t.\exp(-\delta_t)}=\exp(\gamma_t).\frac{a(t-1)}{a(t)} \qquad (4.12)$$

The natural indexation by the growth of salary is corrected by the ratio of two successive annuities.

Remark: fourth design "bis"

This fourth design leads to different pensions for the 2 generations at a same moment [cf. (4.5)]. If we had chosen for the annuity the following value: $a(t)=1+p_t.\exp(-\delta_{t-1})$ then we would have obtained exactly the case of uniform pension developed in section 3 [cf. choice (3. 4)].

4.3.2.2. Model 5 (*life expectancy*)

1°) the revalorization at the total growth of salaries $\qquad R(t)=\exp(\gamma_t+\delta_t)$

2°) the annuity is given by the life expectancy: $\qquad a(t)=1+p_t$

The pension for the old generation is now:

$$\frac{P_2(t)}{P_1(t)}=\frac{\exp(\delta_{t-1})}{p_t}.\left[\exp(\delta_t+\gamma_t).\frac{1+p_t}{\exp(\gamma_t+\delta_t)}-1\right]=e^{\delta_{t-1}} \qquad (4.13)$$

The indexation process becomes:

$$RP(t)=\frac{P_2(t)}{P_1(t-1)}=\exp(\delta_t+\gamma_t).\frac{a(t-1)}{a(t)} \qquad (4.14)$$

4.3.2.3. Model 6 (*revalorization with salary growth*)

1°) the revalorization at the individual growth of salaries: $\qquad R(t)=\exp(\gamma_t)$

2°) the annuity is given by: $\qquad a(t)=1+p_t\, e^{-\delta_t}$

The pension for the old generation is now:

$$\frac{P_2(t)}{P_1(t)}=\frac{\exp(\delta_{t-1})}{p_t}(\exp(\delta_t)+p_t-1) \qquad (4.15)$$

The indexation process has the following more complicated form:

$$RP(t)=\frac{P_2(t)}{P_1(t-1)}=\exp(\delta_{t-1}+\gamma_t).\frac{a(t-1)}{a(t)}.\frac{(e^{\delta_t}+p_t-1)}{p_t} \qquad (4.16)$$

4.4. NON EQUILIBRATED SCHEMES AND STABILIZATION FUND

Other designs could be considered without the equilibrium condition. Then the system will generate each year a deficit or a gain which can be incorporated into a stabilization fund. We denote by $F(t)$ the value of this fund at time t. $F(0)$ will represent the initial amount available in the fund.

Then the recursive evolution of the fund is given by:

$$F(t+1) = (F(t) + C(t) - P(t).L(t))(1+i_t) \qquad (4.17)$$

where i is the return of the fund.

We could for instance think at the following design:

1°) the revalorization process before retirement is equal to the growth of individual salary:

$$R(t) = \exp(\gamma_t)$$

2°) the annuity is given by "its natural value":

$$a(t) = 1 + p_{t+1}\, e^{-\delta_t}$$

3º) the indexation after retirement is the same as in model 1. It is easy to show that this particular choice is no more solution of the equilibrium relation (4.4).

5. CONCLUSION AND FURTHER RESEARCH

In this paper we have developed an overlapping model of pension with 3 generations. A NDC pension system is mainly based on 3 processes: revalorization of salaries, value of the annuity and indexation of the pensions. Based on the classical equation of PAYG, we have obtained an equilibrium equation for these three parameters. A lot of coherent (and different) schemes have been presented. For instance, a basic system (what we have called "*the canonical choice*") consists of the following choice: the revalorization of salaries follows the general evolution of the total salaries; the annuity is computed using prospective probability and observed demographic growth. Then we have showed that a natural indexation of pensions on individual growth of salaries was not equilibrated and has to be corrected by two demographic ratios reflecting the change in longevity. Other combinations of parameters have also been considered. The methodology used in this paper is based on deterministic scenarios. Taking into account the future uncertainty of the three basic parameters, moving to a stochastic modelization will be an important topic to be addressed in the near future.

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