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NDC Dynamic Equilibrium Model with financial and demographic risks

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Outline

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- 2. DEMOGRAPHIC MODEL
- 3. STATIC EQUILIBRIUM
- 4. DYNAMIC UNIFORM PENSION MODEL
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1. NDC PAYG DC or DB…

NDC = Notional Defined Contribution Schemes

= Notional Accounts

- Attempt to reproduce the logic of individual funded pension accounts but now in a PAYG framework.

- Combination of :
  - solidarity of PAYG technique
  - stability of the charges in DC

(*Disney (1999), Scherman (1999), Williamson(2004),…)*

1. NDC PAYG DC or DB…

<u>2 basic techniques in order to finance pension liabilities</u>

PAY AS YOU GO

↓

Pensions for retirees
are paid by active people

Unfunded schemes

FUNDING

↓

Active people finance
their own pension

Funded schemes

1. NDC PAYG DC or DB…

This fundamental choice is present as well for defined benefit pension schemes (**DB**) as for defined contributions pension schemes (**DC**).

	Pay as you go	**Funding**
DB	*Classical Social Security*	*Classical Employee Benefit DB plan*
DC	***Notional Accounts*** *(NDC)*	*Pension saving Accounts*

1. NDC PAYG DC or DB…

By definition funded DC pension schemes are really "defined contribution" !!!

In particular, the actuarial equilibrium is always guaranteed .

NDC are much more delicate …especially in presence of demographic and financial risks (between DC and DB !!!)

(Valdez Prieto (2000), Settergren (2001),Auerbach/Lee (2006))

Therefore the design of the NDC is crucial .

2. DEMOGRAPHIC MODEL

The Overlapping Generation Model (OLG Model):

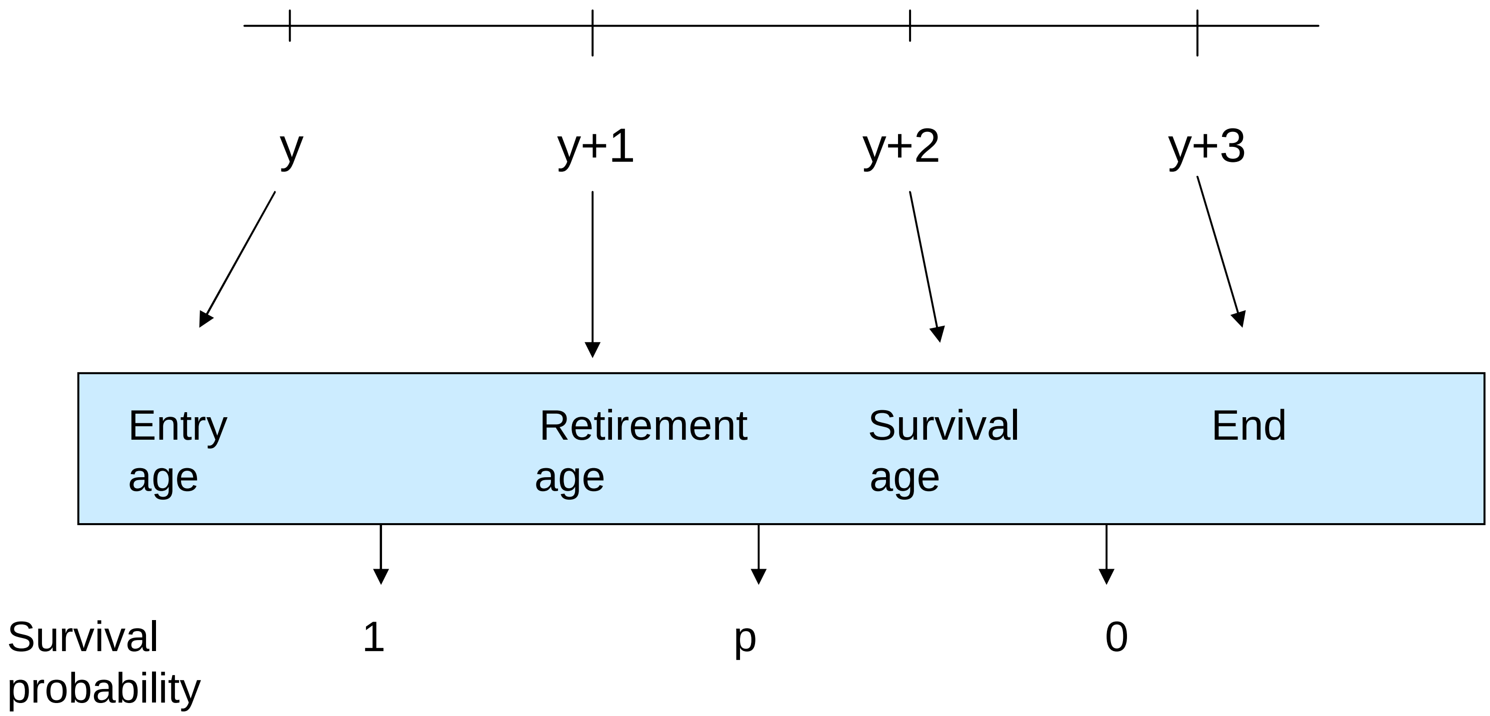
Stylization tool in order to capture the ***dynamic evolution*** of population in time with a focus on equilibrium between active people and retirees. *(for instance Vermeylen (2007))*

OLG Assumptions: model with 3 generations:

- Agents have finite lives
- They live in three periods :
  - they are "active" , then "retired" 2 periods , then dead
  - when one generation becomes old, another young generation is born .

2. DEMOGRAPHIC MODEL

Notations and assumptions : static model :



2. DEMOGRAPHIC MODEL

Notations :

$L(x,t) =$ number of people aged x at time t

$(x = y, y+1, y+2)$

$E(t) = L(y,t) =$ number of people entering at time t

$E(t) = E(0).e^{\delta t}$

mean salary : $s(t) = s(0).e^{\gamma t}$

survival probability between age $y+1$ and $y+2$: p

contribution rate : π

3. STATIC EQUILIBRIUM

PAYG + DC scheme for this population:

Actuarial equivalence between income and outcome

Income :

$$\text{total contributions}: C(t) = \pi.s(t).L(y,t)$$
$$= \pi.s(0).L(y,0).e^{\delta t}.e^{\gamma t}$$

Outcome :

$$\text{total pensions}: \quad P(t).(L(y+1,t)+L(y+2,t))$$
$$= P(t).E(0).(e^{\delta(t-1)} + p.e^{\delta(t-2)})$$

(*same pension for both generations*)

3. STATIC EQUILIBRIUM

Mean pension achieved :

$$P(t) = \frac{\pi.s(0).e^{\gamma(t-1)}.e^{\gamma+\delta}}{1+p.e^{-\delta}}$$

Interpretation as a NDC formula :

$$P(t) = \frac{c(t-1).R(t)}{a(t)} = \frac{\text{sum of contributions with revalorization}}{\text{annuity}}$$

with $c(t-1) = \text{contribution} = \pi.s(0).e^{\gamma(t-1)}$

$R(t) = \text{revalorization} = e^{\gamma+\delta}$

$a(t) = \text{annuity} = 1 + p.e^{-\delta}$

3. STATIC EQUILIBRIUM

Indexation process :

$$RP(t) = \frac{P(t)}{P(t-1)} = e^{\gamma}$$

⟶ Following the indexation of wages

Replacement rate :

$$RR(t) = \frac{P(t)}{s(t)} = \pi . \frac{e^{\delta}}{1 + p\, e^{-\delta}}$$

3. STATIC EQUILIBRIUM

CONCLUSION :

In order to design a NDC scheme , we must define 3 processes :

a) the indexation process (pension indexation after retirement): **RP**

b) the revalorization process of the salaries : **R**

c) the annuity : **a**

3. STATIC EQUILIBRIUM

In the static model, the PAYG constraint gives the following ***canonical identification*** :

$$RP(t) = e^{\gamma}$$ → *Pension Indexation* : salary growth

$$R(t) = e^{\gamma+\delta}$$ → *Revalorization* : salary + population growth

$$a(t) = 1 + p.e^{-\delta}$$ → *Annuity* : population growth

↓

Canonical design of the NDC scheme

3. STATIC EQUILIBRIUM

Remark :

Even if, in this static model, this canonical identification seems natural, a lot of other identifications are possible (giving off course finally the same pension amount !!). In fact, the processes R(t) and a(t) can be multiplied by any positive constant (…presentation purpose…) :

$$P(t) = \frac{c(t-1).(\alpha.R(t))}{\alpha.a(t)}$$

For instance :

Same indexation

$$RP(t) = e^{\gamma}$$
$$R(t) = e^{\gamma+\delta}$$
$$a(t) = 1 + p.e^{-\delta}$$

→

$$RP(t) = e^{\gamma}$$
$$R(t) = RP(t) = e^{\gamma}$$
$$a(t) = e^{-\delta} + p.e^{-2\delta}$$

4. DYNAMIC UNIFORM PENSION

Static model → Dynamic model

δ : growth rate of population	δ_t (year t ; $t = 1,2,\ldots$)
γ: growth rate of wages	γ_t
p: survival probability	p_t

2 cases for the pension amount :
- uniform pension at time t
- 2 generations of retires receiving different pensions

4. DYNAMIC UNIFORM PENSION

Uniform pension – equilibrium condition :

static

$$P(t) = \frac{\pi.s(0).e^{\gamma(t-1)}.e^{\gamma+\delta}}{1+p.e^{-\delta}}$$

dynamic

$$P(t) = \frac{\pi.s(0).\exp(\sum_{i=1}^{t-1}\gamma_i).e^{\gamma_t+\delta_t}}{1+p_t.e^{-\delta_{t-1}}}$$

4. DYNAMIC UNIFORM PENSION

NDC canonical interpretation :

$$P(t) = \frac{\pi . s(0). \exp(\sum_{i=1}^{t-1} \gamma_i).e^{\gamma_t + \delta_t}}{1 + p_t.e^{-\delta_{t-1}}}$$

?

$$P(t) = \frac{c(t-1).R(t)}{a(t)}$$

$$RP(t) = \frac{P(t)}{P(t-1)}$$

$$R(t) = \exp(\gamma_t + \delta_t)$$

$$a(t) = 1 + p_t.e^{-\delta_{t-1}}$$

$$RP(t) = e^{\gamma_t}.\frac{a(t-1)}{a(t)}.\frac{e^{\delta_t}}{e^{\delta_{t-1}}}$$

4. DYNAMIC UNIFORM PENSION

NDC canonical interpretation : conclusions

1°) **R(t): *revalorization of salaries*:**
as expected : salary + population growth

2°) **a(t): *conversion annuity*** :
as expected : population + probability
(… but time related !!!)

3°) **RP(t) : *indexation of the pension :***
not as expected !!!
No more simply equal to salary growth

4. DYNAMIC UNIFORM PENSION

New indexation rule :

$$RP(t) = e^{\gamma_t} . \frac{a(t-1)}{a(t)} . \frac{e^{\delta_t}}{e^{\delta_{t-1}}}$$

- e^{γ_t}: Salary Growth
- $\frac{a(t-1)}{a(t)}$: Demographic and longevity risk
- $\frac{e^{\delta_t}}{e^{\delta_{t-1}}}$: Demographic risk

The expected indexation coefficient is corrected by 2 terms :

- *a ratio of successive annuities*
- *a ratio of successive demographic growths*

4. DYNAMIC UNIFORM PENSION

Other possible NDC identifications :

EXAMPLE 1 : *uniform indexation before and after retirement :*

$$RP(t) = e^{\gamma_t} . \frac{a(t-1)}{a(t)} . \frac{e^{\delta_t}}{e^{\delta_{t-1}}} = R(t)$$

$$\longrightarrow \quad a(t) = e^{-\delta_{t-1}} + p_{t-1}.e^{-(\delta_{t-1}+\delta_{t-2})}$$

EXAMPLE 2 : *revalorization in line with salary growth :*

$$R(t) = e^{\gamma_t}$$

$$\longrightarrow \quad a(t) = e^{-\delta_t} + p_t.e^{-(\delta_t+\delta_{t-1})}$$

5. DYNAMIC GENERATION PENSION

$$P(t) = \text{uniform pension}$$

↓

$$P_1(t) = \text{pension for the new retired generation}$$
$$P_2(t) = \text{pension for the old retired generation}$$

<u>Equilibrium condition :</u>

$$L(y, t).\pi.s(t) = L(y+1, t).P_1(t) + L(y+2, t).P_2(t)$$

….Infinite number of different equilibrated schemes !!!

5. DYNAMIC GENERATION PENSION

Additional constraint : NDC definition :

The first pension is computed using at retirement a NDC formula .
The second pension is then a consequence of the equilibrium relation.

Different NDC identifications will now lead to different pension amounts (balance between the 2 generations) !!!

General NDC approach :

The pension for the new generation is given by :

$$P_1(t) = \frac{c(t-1).R(t)}{a(t)}$$

→ The processes R(t) and a(t) must be chosen.

5. DYNAMIC GENERATION PENSION

Then the pension for the old generation is given by the equilibrium condition :

$$\frac{P_2(t)}{P_1(t)} = \frac{C(t)}{L(y+2,t)} \cdot \frac{1}{P_1(t)} - \frac{L(y+1,t)}{L(y+2,t)}$$

The indexation process is given by the relation :

$$RP(t) = \frac{P_2(t)}{P_1(t-1)}$$

5. DYNAMIC GENERATION PENSION

Canonical choice : (model 1):

$$R(t) = \exp(\gamma_t + \delta_t)$$

Natural revalorization

$$a(t) = 1 + p_{t+1}.e^{-\delta_t}$$

Annuity with Prospective Life table

$$RP(t) = e^{\gamma_t} . \frac{p_{t+1}}{p_t} . \frac{a(t-1)}{a(t)}$$

Natural Indexation

Longevity correction

Annuity correction

5. DYNAMIC GENERATION PENSION

Other possible choices : (model 2):

$$R(t) = \exp(\gamma_t + \delta_t)$$

$$a(t) = 1 + p_{t+1}$$

Natural revalorization

Annuity Life Expectancy

$$RP(t) = e^{\delta_t + \gamma_t} \cdot \frac{p_{t+1}}{p_t} \cdot \frac{a(t-1)}{a(t)}$$

Indexation = revalorization

Longevity correction

Annuity correction

5. DYNAMIC GENERATION PENSION

Other possible choices : (model 4):

$$R(t) = \exp(\gamma_t + \delta_t)$$

$$a(t) = 1 + p_t e^{-\delta_t}$$

Natural revalorization

Annuity with present Life table

$$RP(t) = e^{\gamma_t} . \frac{a(t-1)}{a(t)}$$

Natural Indexation

Annuity correction

6. CONCLUSION

In a dynamic model the indexation process after retirement can not be chosen so naturally.

Different choices can be done with respect to the annuity conversion and the revalorization of salaries.

Then in an equilibrium model, the indexation after retirement becomes a constraint and is generally not equal to a natural indexation . Correction terms mainly based on an annuity ratio appear systematically.

FURTHER RESEARCH

- Extension in a more sophisticated demographic model with "many" ages;

- Stochastic approach for the financial and demographic background;

- Introduction of an Equilibrium Fund in this stochastic framework.

THANK YOU

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